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Recursion

Readings:
Zybooks Chapter 16

Slides are modified from those provided by Marty Stepp
www.buildingjavaprograms.com
Some materials from Joseph Hollingsworth and Murali Sitaraman

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Recursion

- **Recursion**: The definition of an operation in terms of itself.
  - Solving a problem using recursion depends on solving smaller occurrences of the same problem.
- **Recursive Programming**: Writing methods that call themselves to solve problems recursively.
  - An equally powerful substitute for *iteration* (loops)
  - Particularly well-suited to solving certain types of problems

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Why learn recursion?

- Some important data structures are inherently recursive (e.g., lists and search trees: coming soon!).
- Some algorithms are inherently recursive (e.g., "divide and conquer" algorithms like mergesort and binary search: also coming soon!).
- Many "functional" programming languages (e.g., Scheme, ML, and Haskell) use recursion exclusively (no loops).
- Leads to elegant, simplistic, short code (when used well).

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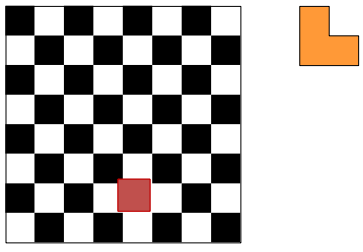
Example: Trominos Puzzle

- Suppose you are given a $2^i \times 2^i$ chessboard, for some integer $i \geq 1$, with a single tile already covered (it could be any tile, you don't get to choose).
- Can you cover the remaining tiles exclusively using "trominos" (three tiles in an L shape)?
- The answer is yes, but how?



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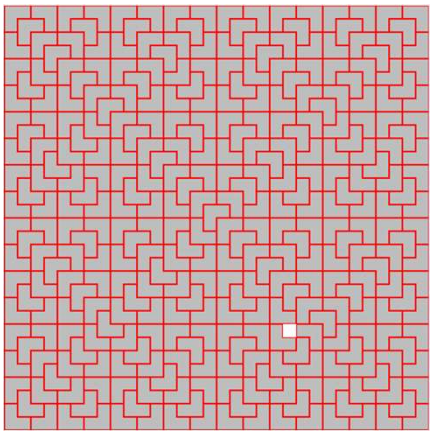
Example: Trominos Puzzle

- For example, here is a solution to a $2^5 \times 2^5$ board.



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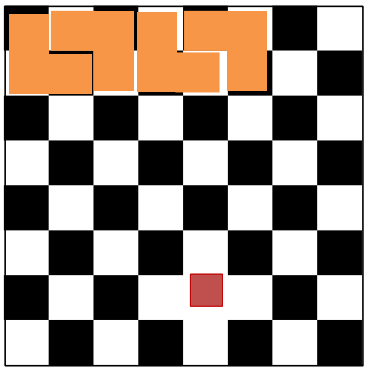
Example: Trominos Puzzle



- Looking at a $2^3 \times 2^3$ board, you might be tempted to just iterate across the board.
- But how do you know you won't get stuck later?
- And how would you program all of the possible cases, depending on where the covered tile is?

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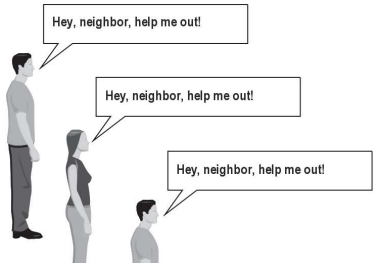
The idea

- Recursion is all about breaking a big problem into smaller occurrences of that same problem.
  - Each person can solve a small part of the problem.
    - What is a small version of the problem that would be easy to answer?
    - What information from a neighbor might help me?



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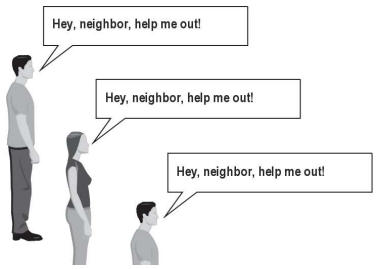
Recursive algorithm

- Number of people behind me:
  - If there is someone ahead of me,
    ask them how many people are in front of them.
    - When they respond with a value **N**, then I will answer **N + 1**.
  - If there is no one ahead of me, I will answer **0**.



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Recursion and cases

- Every recursive algorithm involves at least 2 cases (these can be implicit):
  - **Base Case**: A simple occurrence that can be answered directly.
  - **Recursive Case**: A more complex occurrence of the problem that cannot be directly answered, but can instead be described in terms of smaller occurrences of the same problem.

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Recursion and cases

- Every recursive algorithm involves at least 2 cases (these can be implicit):
  - **Base Case**: A simple occurrence that can be answered directly.
  - **Recursive Case**: A more complex occurrence of the problem that cannot be directly answered, but can instead be described in terms of smaller occurrences of the same problem.
- Some recursive algorithms have more than one base or recursive case, but all have at least one of each.
- A crucial part of recursive programming is identifying these cases.

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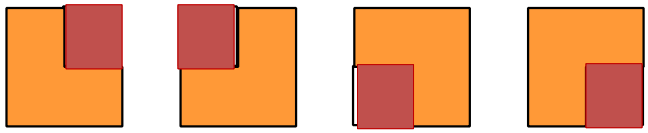
Trominos Puzzle - Recursive Tiling

- Thinking back to our chessboard and trominos, the base case of a $2^1 \times 2^1$ board with a single tile covered is easy: there are exactly four possibilities.



- For the recursive case, we want to tile a $2^i \times 2^i$ board (with a single tile covered) by breaking it up into four different $2^{i-1} \times 2^{i-1}$ boards (each with a single tiled covered).

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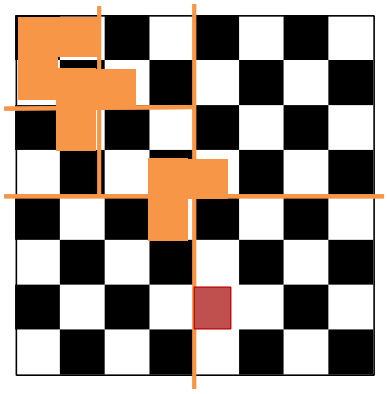
Trominos Puzzle - Recursive Tiling



- That just means splitting the chessboard into the quadrants.
- To ensure that every subproblem has a single tile covered, we can place a single tromino in the center.
- That's the entire recursive algorithm! But how do we actually program recursively?

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Recursive Programming – Big Ideas

- A recursive function calls itself in its definition.
  - Suppose we have a function called **`function()`**
- To end in finite time, linear time, we should be parameterize the function by problem size, like **`n`** (e.g., **`function(n)`**):
  - First check for the base case (when **`n`** is small), and return the answer **without** recursive calls.
  - Otherwise, make one or more calls to **`function(m)`** where **`m`** < **`n`**, in order to return the solution to **`function(n)`**.

```
int function(int n) {
   if (…) { return; } //base case
   else { return fun(n-1); } //recursive case
}
```


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Recursion in Java

- Consider the following method to print a line of * characters:

```
// Prints a line containing the given number of stars.
// Precondition: n >= 0
public static void printStars(int n) {
    for (int i = 0; i < n; i++) {
        System.out.print("*");
    }
    System.out.println();   // end the line of output
}
```

- Write a recursive version of this method (that calls itself).
  - Solve the problem without using any loops.

A base case

- What are the cases to consider?
  - What is a very easy number of stars to print without a loop?

```
public static void printStars(int n) {
    if (n == 1) {
        // base case; just print one star
        System.out.println("*");
    } else {
        ...
    }
}
```


Handling more cases

- Handling additional cases, with no loops (in a bad way):

```
public static void printStars(int n) {
    if (n == 1) {
        // base case; just print one star
        System.out.println("*");
    } else if (n == 2) {
        System.out.print("*");
        System.out.println("*");
    } else if (n == 3) {
        System.out.print("*");
        System.out.print("*");
        System.out.println("*");
    } else if (n == 4) {
        System.out.print("*");
        System.out.print("*");
        System.out.print("*");
        System.out.println("*");
    } else ...
}
```

Handling more cases 2

- Taking advantage of the repeated pattern (somewhat better):

```
public static void printStars(int n) {
    if (n == 1) {
        // base case; just print one star
        System.out.println("*");
    } else if (n == 2) {
        System.out.print("*");
        printStars(1);    // prints "*"
    } else if (n == 3) {
        System.out.print("*");
        printStars(2);    // prints "**"
    } else if (n == 4) {
        System.out.print("*");
        printStars(3);    // prints "***"
    } else ...
}
```


Using recursion properly

- Condensing the recursive cases into a single case:

```
public static void printStars(int n) {
    if (n == 1) {
        // base case; just print one star
        System.out.println("*");
    } else {
        // recursive case; print one more star
        System.out.print("*");
        printStars(n - 1);
    }
}
```

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"Recursion Zen"

- The real, even simpler, base case is an `n` of 0, not 1:

```
public static void printStars(int n) {
    if (n == 0) {
        // base case; just end the line of output
        System.out.println();
    } else {
        // recursive case; print one more star
        System.out.print("*");
        printStars(n - 1);
    }
}
```

  - **Recursion Zen**: The art of properly identifying the best set of cases for a recursive algorithm and expressing them elegantly.

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Extra recursive trace

- Consider the following recursive method:

```
public static int mystery(int n) {
    if (n < 10) {
        return n;
    } else {
        int a = n / 10;
        int b = n % 10;
        return mystery(a + b);
    }
}
```

  - What is the result of the following call?

```
mystery(648)
```

A recursive trace

- Consider the following recursive method:

```
public static int mystery2(int n) {
    if (n < 10) {
        return (10 * n) + n;
    } else {
        int a = mystery2(n / 10);
        int b = mystery2(n % 10);
        return (100 * a) + b;
    }
}
```

  - What is the result of the following call?

```
mystery2(348)
```


Exercise

- Write a recursive method `pow` accepts an integer base and exponent and returns the base raised to that exponent.
  - Example: `pow(3, 4)` returns 81
  - Solve the problem recursively and without using loops.

An optimization

- Notice the following mathematical property:

$3^{12} = 531441 = 9^6$

$= (3^2)^6$

$531441 = (9^2)^3$

$= ((3^2)^2)^3$

  - When does this "trick" work?
  - How can we incorporate this optimization into our `pow` method?
  - What is the benefit of this trick if the method already works?

pow solution 2

```
// Returns base ^ exponent.
// Precondition: exponent >= 0
public static int pow(int base, int exponent) {
    if (exponent == 0) {
        // base case; any number to 0th power is 1
        return 1;
    } else if (exponent % 2 == 0) {
        // recursive case 1:  x^y = (x^2)^(y/2)
        return pow(base * base, exponent / 2);
    } else {
        // recursive case 2:  x^y = x * x^(y-1)
        return base * pow(base, exponent - 1);
    }
}
```

Verification of Recursion

- How do we know a recursive method works?
  - Static analysis (static)
  - Testing (dynamic)
  - Formal verification (static)
- Formal Verification
  - Correctness proofs using principles learned in Discrete Math
    - Shows correctness of the algorithm on all valid inputs
  - What type of proof strategy is recursion similar to?

Verification of Recursion

- When proving an operation implementation is correct, the verifier has:
  - Preconditions: assume true at the beginning of the method (initial state(s))
  - Post Conditions (Ensures/Goal): what you must prove to be true at the end of the method (final state(s))

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Verification of Recursion

- When proving a call of an operation is correct
  - Preconditions: prove that the input(s) to the operation is(are) correct before calling the operation
  - Post Conditions: assume operation's post conditions are true after the call

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Method of Proof by Mathematical Induction
[Epp]

Consider a statement of the form, "For all integers, n>=a, a property P(n) is true." To prove such a statement, perform the following steps:

Step 1 (basis step): Show that the property is true for n=a.

Step 2 (inductive step): Show that for all integers m>=a, if the property is true for n=m then it is true for n=m+1. To perform this step,

suppose that the property is true for n=m, where m is any particular but arbitrarily chosen integer with m>=a. [*This supposition is called the* ***inductive hypothesis***.]

Then **show** that the property is true for n=m+1.

Example

[Based on code from RESOLVE Project]

```
// Precondition: j >= 0
// Goal/Ensures:
//     result = j + k
int sum(int j, int k) {
   if (j == 0) {
      return k;
   } else {
      int r = sum(j - 1, k);
      return r + 1;
   }
}
```


Example

[Based on code from RESOLVE Project]

For all integers j>=0, sum(j, k) = j+k.

```
// Precondition: j >= 0
// Goal/Ensures:
//     result = j + k
int sum(int j, int k) {
   if (j == 0) {
      return k;
   } else {
      int r = sum(j - 1, k);
      return r + 1;
   }
}
```

Example

[Based on code from RESOLVE Project]

```
// Precondition: j >= 0
// Goal/Ensures:
//     result = j + k
int sum(int j, int k) {
   if (j == 0) {
      return k;
   } else {
      int r = sum(j - 1, k);
      return r + 1;
   }
}
```

For all integers j>=0, sum(j, k) = j+k.

Step 1 (basis step): Show that the property is true for j=0.

Example

[Based on code from RESOLVE Project]

```
// Precondition: j >= 0
// Goal/Ensures:
//     result = j + k
int sum(int j, int k) {
   if (j == 0) {
      return k;
   } else {
      int r = sum(j - 1, k);
      return r + 1;
   }
}
```

For all integers j>=0, sum(j, k) = j+k.

Step 1 (basis step): Show that the property is true for j=0.

Assume j=0. We must show that sum(0, k) = 0 +k.

Based on the code, sum(0, k) returns k.
k = 0 + k

Therefore true for j = 0.

Example

[Based on code from RESOLVE Project]

```
// Precondition: j >= 0
// Goal/Ensures:
//     result = j + k
int sum(int j, int k) {
    if (j == 0) {
        return k;
    } else {
        int r = sum(j - 1, k);
        return r + 1;
    }
}
```

For all integers j>=0, sum(j, k) = j+k.

Step 2 (inductive step): Show that for all integers m>=0, if the property is true for j=m then it is true for j=m+1.

suppose that the property is true for j=m, where j is any particular but arbitrarily chosen integer with m>=0.

Then **show** that the property is true for j=m+1. [sum(m+1, k) = (m+1) + k]

Example

[Based on code from RESOLVE Project]

```
// Precondition: j >= 0
// Goal/Ensures:
//     result = j + k
int sum(int j, int k) {
    if (j == 0) {
        return k;
    } else {
        int r = sum(j - 1, k);
        return r + 1;
    }
}
```

For all integers j>=0, sum(j, k) = j+k.

Step 2 (inductive step): Show that for all integers m>=0, if the property is true for j=m then it is true for j=m+1.

suppose that the property is true for j=m, where j is any particular but arbitrarily chosen integer with m>=0.
sum(m, k) = m+k

Then **show** that the property is true for j=m+1. [sum(m+1, k) = (m+1) + k]
sum(m+1, k) = sum(m, k) + 1

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Example

[Based on code from RESOLVE Project]

```
// Precondition: j >= 0
// Goal/Ensures:
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int sum(int j, int k) {
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    } else {
        int r = sum(j - 1, k);
        return r + 1;
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}
```

For all integers j>=0, sum(j, k) = j+k.

Step 2 (inductive step): Show that for all integers m>=0, if the property is true for j=m then it is true for j=m+1.

suppose that the property is true for j=m, where j is any particular but arbitrarily chosen integer with m>=0.
sum(m, k) = m+k

Then **show** that the property is true for j=m+1. [sum(m+1, k) = (m+1) + k]
sum(m+1, k) = sum(m, k) + 1

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Example

[Based on code from RESOLVE Project]

```
// Precondition: j >= 0
// Goal/Ensures:
//     result = j + k
int sum(int j, int k) {
    if (j == 0) {
        return k;
    } else {
        int r = sum(j - 1, k);
        return r + 1;
    }
}
```

For all integers j>=0, sum(j, k) = j+k.

Step 2 (inductive step): Show that for all integers m>=0, if the property is true for j=m then it is true for j=m+1.

suppose that the property is true for j=m, where j is any particular but arbitrarily chosen integer with m>=0.
sum(m, k) = m+k

Then **show** that the property is true for j=m+1. [sum(m+1, k) = (m+1) + k]
sum(m+1, k) = sum(m, k) + 1
= (m+k) + 1 // based on
// assumption

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Example

[Based on code from RESOLVE Project]

```
// Precondition: j >= 0
// Goal/Ensures:
//     result = j + k
int sum(int j, int k) {
    if (j == 0) {
        return k;
    } else {
        int r = sum(j - 1, k);
        return r + 1;
    }
}
```

For all integers j>=0, sum(j, k) = j+k.

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suppose that the property is true for j=m, where j is any particular but arbitrarily chosen integer with m>=0.
sum(m, k) = m+k

Then **show** that the property is true for j=m+1. [sum(m+1, k) = (m+1) + k]
sum(m+1, k) = sum(m, k) + 1
= (m+k) + 1 // based on // assumption
= (m+1) + k

Therefore true for j>=0.

Exercise

- Write a recursive method `generateBinary` that accepts an integer and returns that number's representation in binary (base 2).
  - Example: `generateBinary (7)` returns 111
  - Example: `generateBinary (12)` returns 1100
  - Example: `generateBinary (42)` returns 101010

place	10	1
value	**4**	**2**

32	16	8	4	2	1
1	**0**	**1**	**0**	**1**	**0**

Slides are modified from those provided by Marty Stepp

```java
/**
 * Converts a decimal number to binary
 *
 * @param decimal number to convert
 * @return binary string of number
 * @throws IllegalArgumentException if decimal < 0
 */
public static String convertToBinary(int decimal) {
  if (decimal < 0) {
     throw new IllegalArgumentException("Negative value");
  }
  String binary = "";
  if (decimal == 0) { // Special case
     return "0";
  }
  while (decimal != 0) {
     binary = (decimal % 2) + binary;
     decimal /= 2;
  }
  return binary;
}
```


Case analysis

- Recursion is about solving a small piece of a large problem.
  - What is 69743 in binary?
    - Do we know *anything* about its representation in binary?
  - Case analysis:
    - What is/are easy numbers to print in binary?
    - Can we express a larger number in terms of a smaller number(s)?
  - Suppose we are examining some arbitrary integer `N`.
    - if `N`'s binary representation is `10010101011`
    - `(N / 2)`'s binary representation is `1001010101`
    - `(N % 2)`'s binary representation is `1`

Exercise

- Write a recursive method `isPalindrome` accepts a `String` and returns `true` if it reads the same forwards as backwards.
  - `isPalindrome("madam")` → `true`
  - `isPalindrome("racecar")` → `true`
  - `isPalindrome("step on no pets")` → `true`
  - `isPalindrome("able was I ere I saw elba")` → `true`
  - `isPalindrome("Java")` → `false`
  - `isPalindrome("rotater")` → `false`
  - `isPalindrome("byebye")` → `false`
  - `isPalindrome("notion")` → `false`

Exercise

- Write a method `crawl` accepts a `File` parameter and prints information about that file.
  - If the `File` object represents a normal file, just print its name.
  - If the `File` object represents a directory, print its name and information about every file/directory inside it, indented.

```
csc216
    handouts
        syllabus.doc
        lecture_schedule.xls
    homework
        1-sortedintlist
            ArrayIntList.java
            SortedIntList.java
            index.html
            style.css
```

  - **recursive data**: A directory can contain other directories.

`File` objects

- A `File` object (from the `java.io` package) represents a file or directory on the disk.

Constructor/method	Description
`File(`**String**`)`	creates `File` object representing file with given name
`canRead()`	returns whether file is able to be read
`delete()`	removes file from disk
`exists()`	whether this file exists on disk
`getName()`	returns file's name
`isDirectory()`	returns whether this object represents a directory
`length()`	returns number of bytes in file
`listFiles()`	returns a `File[]` representing files in this directory
`renameTo(`**File**`)`	changes name of file

Public/private pairs

- We cannot vary the indentation without an extra parameter:

```
private static void crawl(File f, String indent) {
```

- Often the parameters we need for our recursion do not match those the client will want to pass.

In these cases, we instead write a pair of methods:

1) a public, non-recursive one with the parameters the client wants: `public static void crawl(File f)`
2) a private, recursive one with the parameters we really need: `private static void crawl(File f, String indent)`

References

- Susanna S. Epp, *Discrete Mathematics with Applications*, Third Edition
- Material provided by Joseph Hollingsworth (IUS) and Murali Sitaraman (Clemson) as part of RESOLVE project